CHAPTER

8 Work, Power and Energy

Work Done by constant Force

$$W = \vec{F}\cdot\vec{S}$$

Work Done by Multiple Forces

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

$$W = [\Sigma\vec{F}]\cdot\vec{S}$$

$$W = \vec{F}_1\cdot\vec{S} + \vec{F}_2\cdot\vec{S} + \vec{F}_3\cdot\vec{S} + \cdots \text{ or } W = W_1 + W_2 + W_3 + \ldots$$

Work Done by Variable Force

$$W = \int dW = \int \vec{F}\cdot d\vec{s}$$

Area under the force and displacement curve gives work done.

Important cases:

(i) The force is perpendicular to velocity at all times. The work done in this case is zero. e.g., :

(*a*) A particle attached to a string whose other end is attached to a fixed point. The work done by tension in the string is zero.

(*b*) A body moving on a fixed surface. The work done by normal reaction is zero in this case.

(ii) Work done by gravity on a body near the surface of earth

$= -mg(h_f - h_i)$

$= -mg \times$ increase in height

(iii) Work done by friction on a block moving on a fixed surface

$= -\mu_k \int N(s)ds$

$= -\mu_k N \times$ distance travelled (if N is constant).

Here N is the normal reaction between the block and the surface on which it is moving.

(iv) Work done by a spring on a block

$$= -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

where x_i and x_f refer to the initial and final elongation or compression of the spring.

Relation between Momentum and Kinetic Energy

$$K = \frac{P^2}{2m} \text{ and } P = \sqrt{2mK}$$

P = linear momentum

Potential Energy

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} F\cdot dr$$

i.e., $U_f - U_i = -\int_{r_i}^{r_f} \vec{F}\cdot d\vec{r} = -W_{\text{Conservative}}$

Conservative Forces

$$\vec{F} = -\frac{dU}{dr}\hat{r}$$

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

Equilibrium Conditions

Stable Equilibrium

$$F(r) = -\frac{dU}{dr} = 0; \text{and } \frac{dF}{dr} < 0; \text{ or } \frac{d^2U}{dr^2} > 0$$

Unstable Equilibrium

$$F(r) = -\frac{dU}{dr} = 0; \text{ and } \frac{dF}{dr} > 0; \text{ or } \frac{d^2U}{dr^2} < 0$$

Neutral Equilibrium

$$F(r) = -\frac{dU}{dr} = 0; \text{ and } \frac{dF}{dr} = 0; \text{ or } \frac{d^2U}{dr^2} = 0$$

Work-Energy Theorem for a Particle

$$W_{\text{Net}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Work-Energy Theorem for a System

$$\Delta K + \Delta U = W_{\text{ext}} + W_{\text{int-}NC}$$

Law of Conservation of Mechanical Energy

If the net external force acting on a system is zero and internal non-conservative forces absent, then the mechanical energy is conserved.

$$K_f + U_f = K_i + U_i$$

Power

The average power delivered by an agent is given by

$$P_{\text{avg}} = \frac{W}{t}$$

$$P = \frac{d\vec{F}\cdot\vec{S}}{dt} = \vec{F}\cdot\frac{d\vec{S}}{dt} = \vec{F}\cdot\vec{v}$$